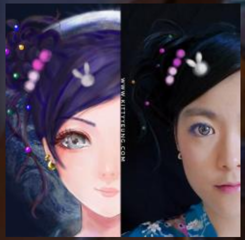
Introduction to Quantum Computing



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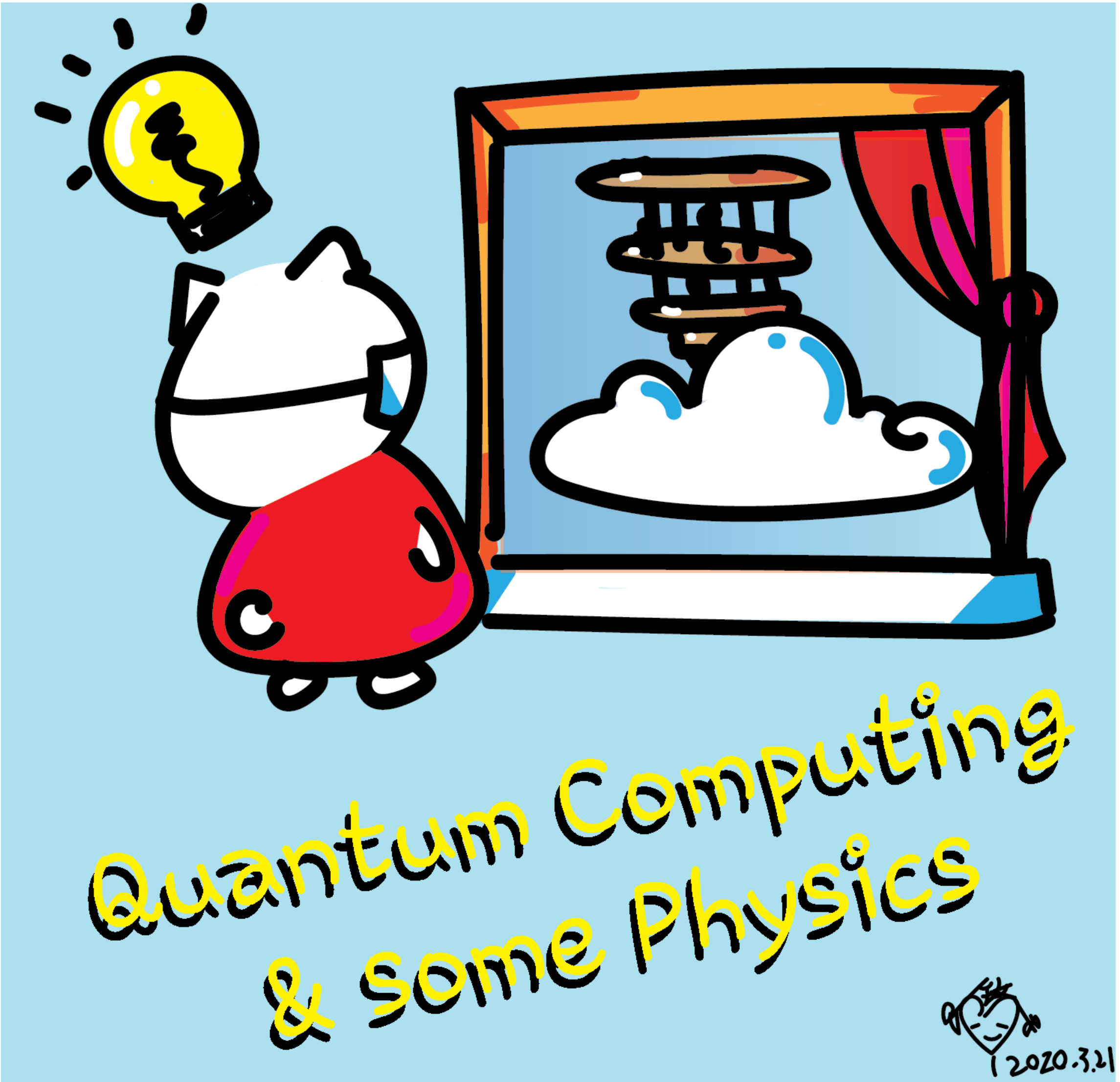
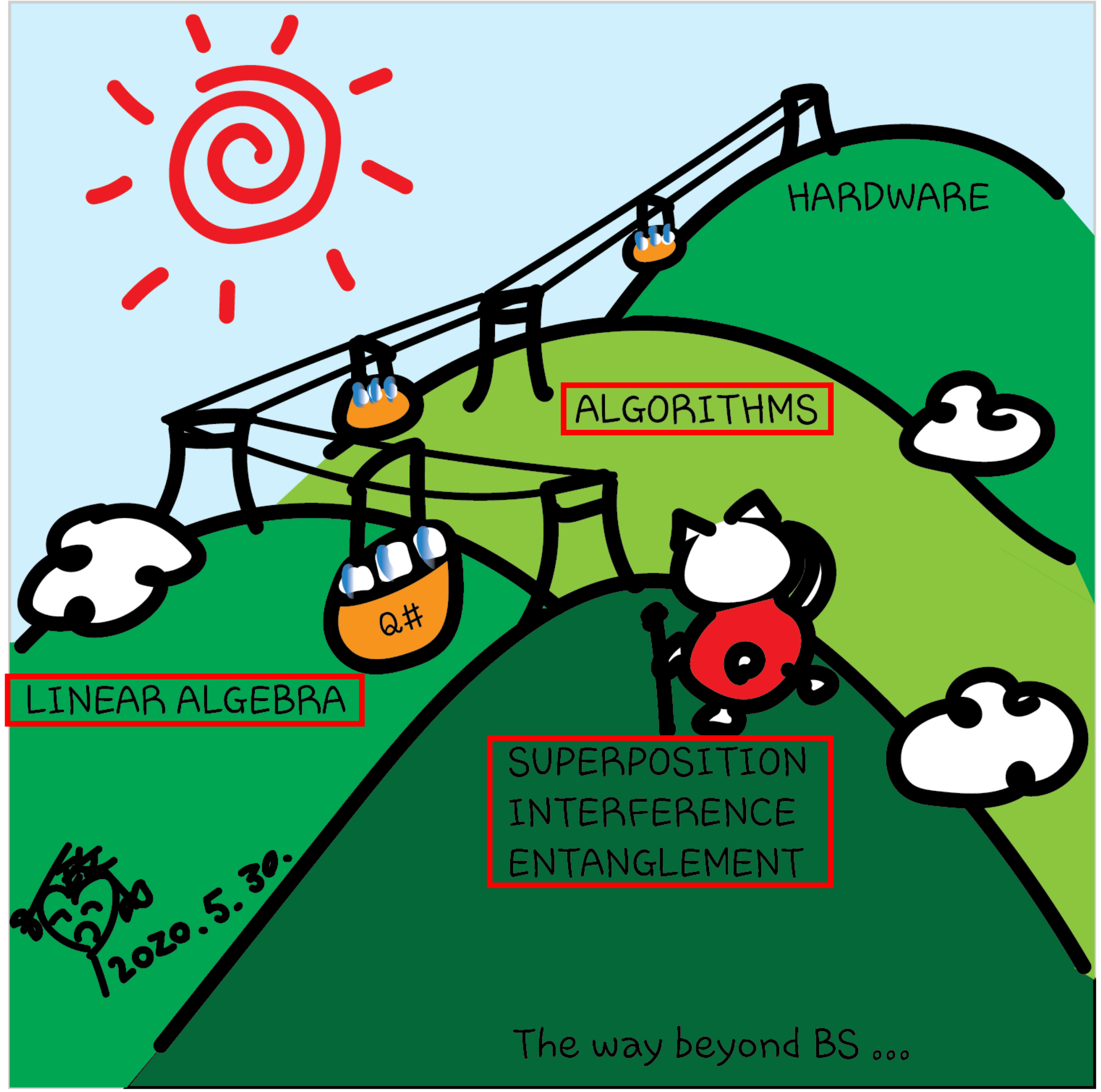
July 19, 2020
Hackaday, session 15
Other communities, session 7

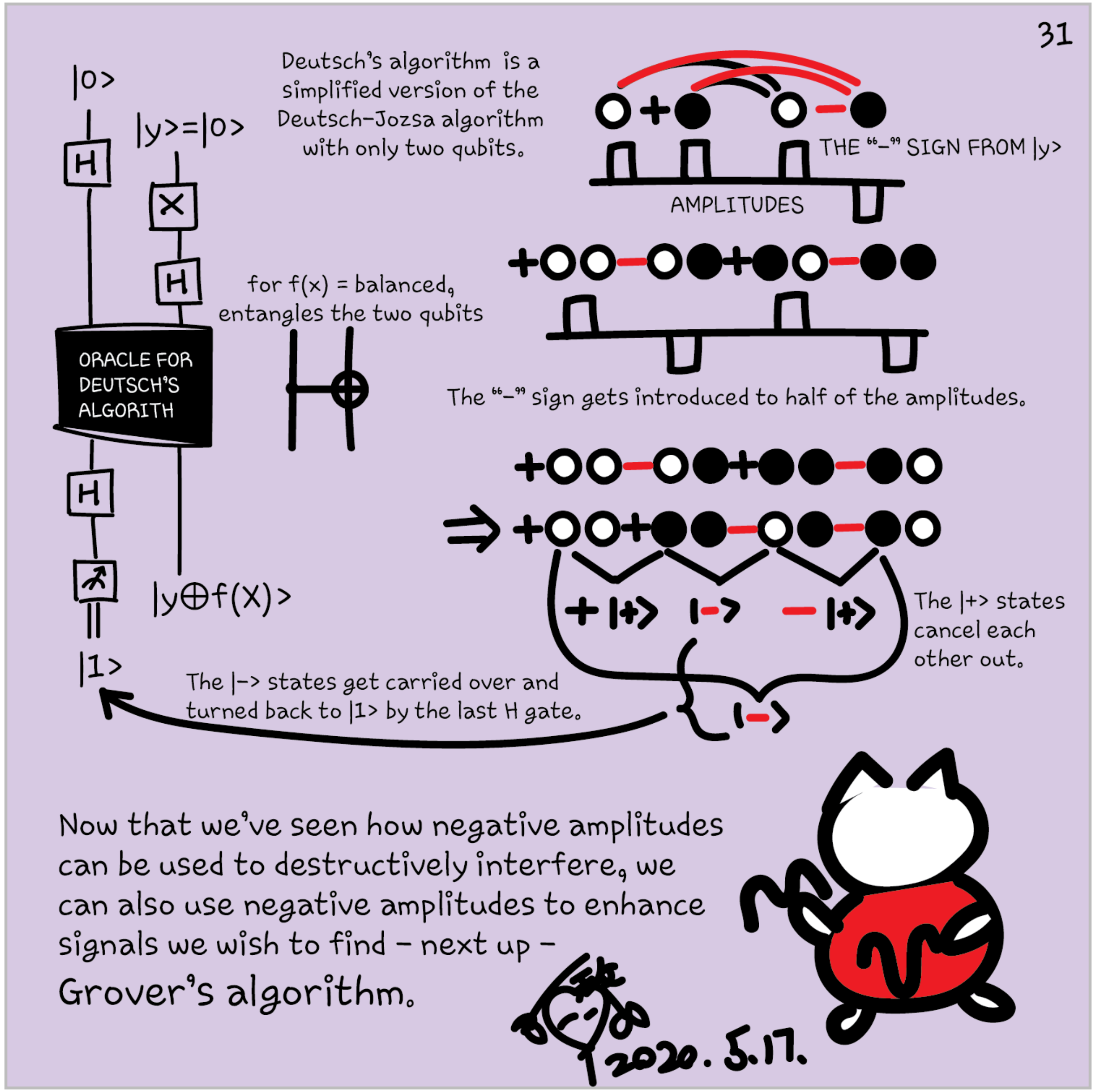
Class structure

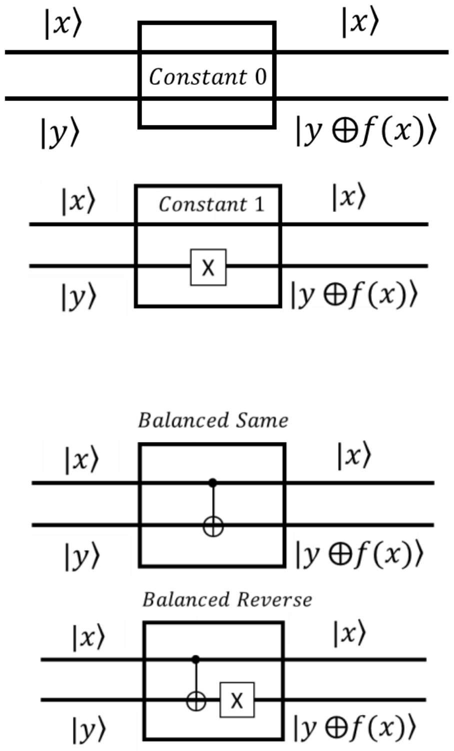
- [Comics on Hackaday – Introduction to Quantum Computing] every Sun
- 30 mins – 1 hour every Sun, one concept (theory, hardware, programming), Q&A
- Contribute to Q# documentation http://docs.microsoft.com/quantum
- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/
- Discuss in Hackaday project comments throughout the week
- Take notes





HARDWARE
ALGORITHMS
Q#
LINEAR ALGEBRA
SUPERPOSITION
INTERFERENCE
ENTANGLEMENT
2020.5.30.
The way beyond BS ...

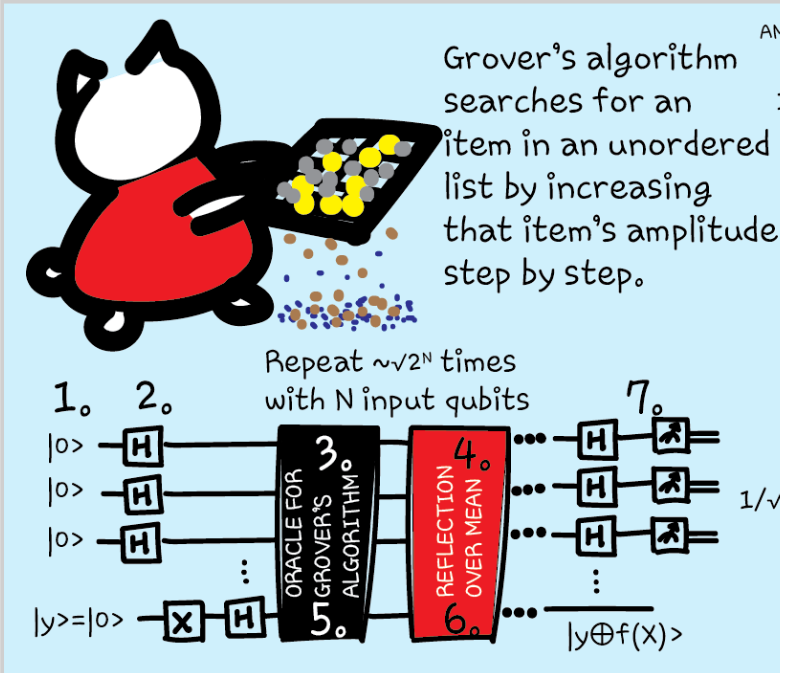

31
Deutsch's algorithm is a simplified version of the Deutsch-Jozsa algorithm with only two qubits.
|0>
|y>=|0>
H
X
H
ORACLE FOR DEUTSCH'S ALGORITH
H
|1>
|y⊕f(X)>
THE "–" SIGN FROM |y>
AMPLITUDES
for f(x) = balanced, entangles the two qubits
The "–" sign gets introduced to half of the amplitudes.
The |+> states cancel each other out.
+ |+>
|–>
– |+>
|–>
The |–> states get carried over and turned back to |1> by the last H gate.
Now that we've seen how negative amplitudes can be used to destructively interfere, we can also use negative amplitudes to enhance signals we wish to find – next up –
Grover's algorithm.
2020.5.17.

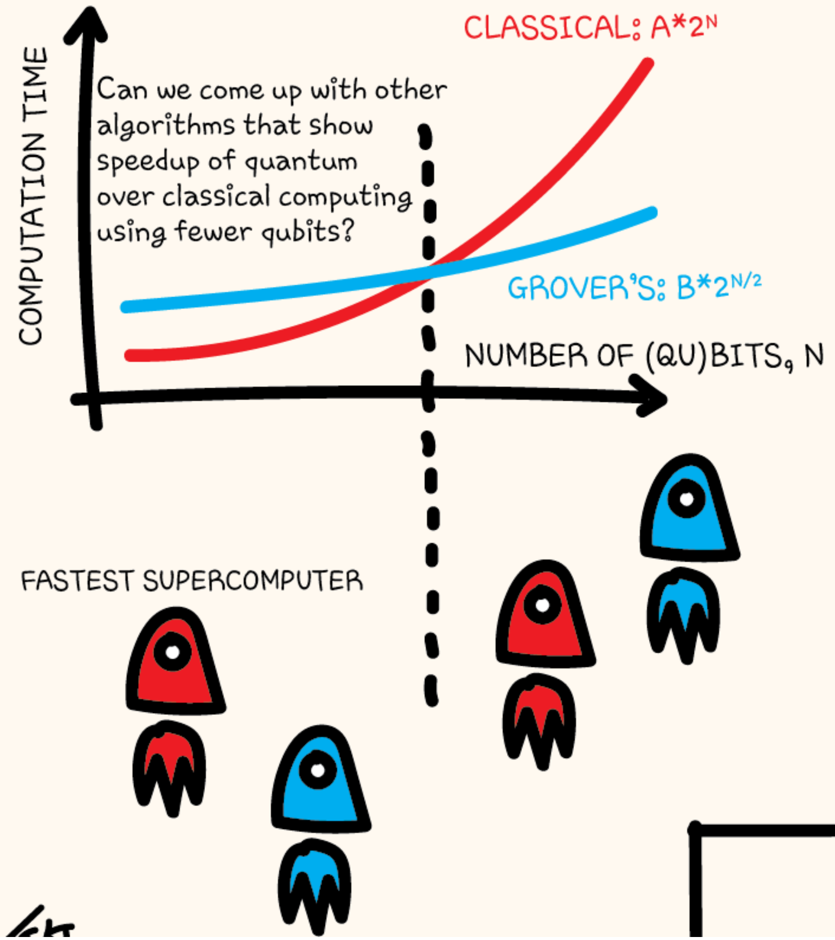

|x⟩
Constant 0
|y⟩
|y ⊕f(x)⟩
|x⟩
Constant 1
X
|y⟩
|y ⊕f(x)⟩
Balanced Same
|x⟩
|y⟩
|y ⊕f(x)⟩
Balanced Reverse
|x⟩
X
|y⟩
|y ⊕f(x)⟩

Grover's algorithm (May 17 Session 8)

https://en.wikipedia.org/wiki/Grover%27s_algorithm

Lov Kumar Grover (* 1960 in Merath, India) is an Indian-American computer scientist





Grover's algorithm provides a speedup over classical algorithms for searching for an item in an unordered list (after a certain number of qubits are reached). A and B are factors that don't depend on N. (They describe how long it takes for the computers to complete the task for a fixed N).

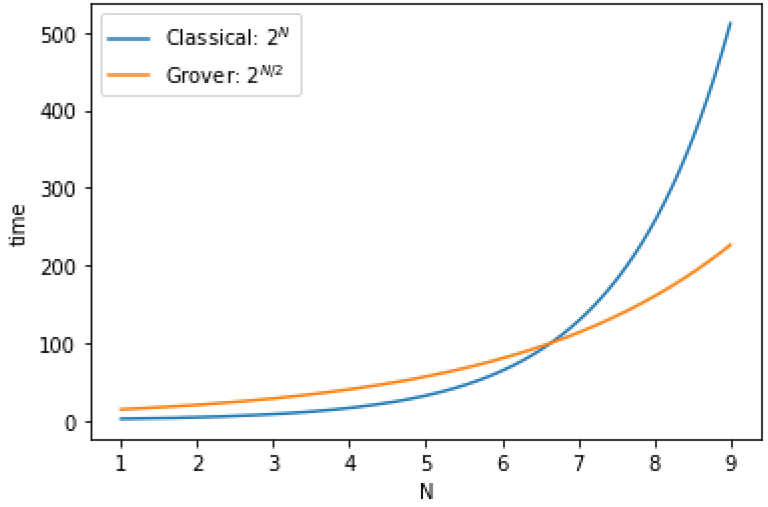
The reflection over the |000...0> state surrounded by H gates is the reflection over mean.

```
[ ] import matplotlib.pyplot as plt
    import numpy as np
```

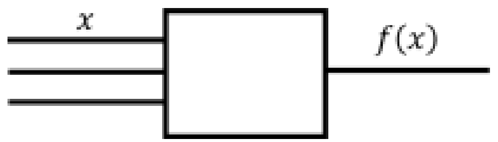
```
[ ] x = np.linspace(1, 9, 1000)
    y = 2**x
    z = 10 * np.sqrt(2**x)
```

```
[ ] plt.plot(x, y, label=r'Classical: $2^N$')
    plt.plot(x, z, label=r'Grover: $2^{N/2}$')
    plt.legend()
    plt.xlabel('N')
    plt.ylabel('time')
```

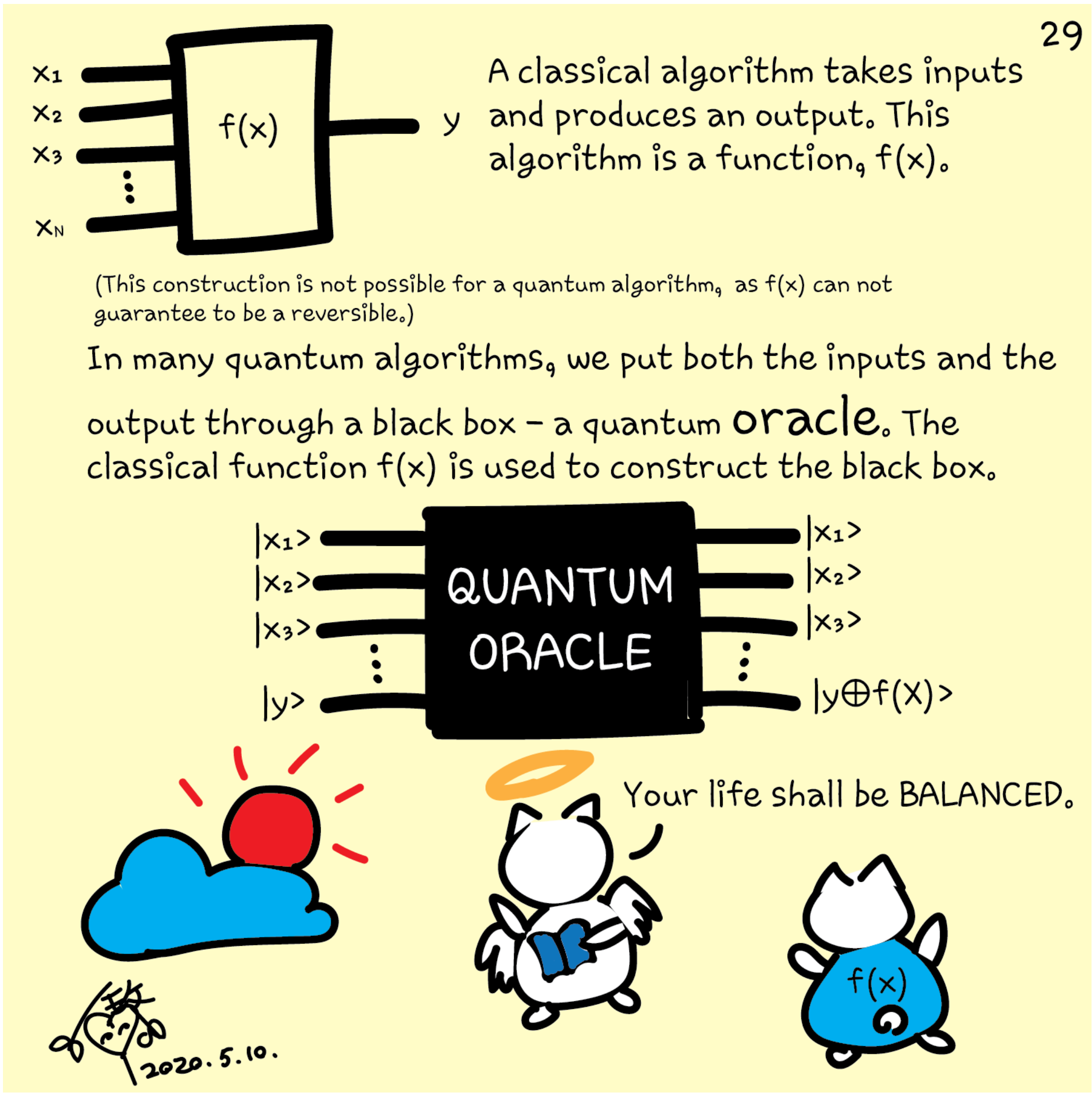
```
Text(0, 0.5, 'time')
```



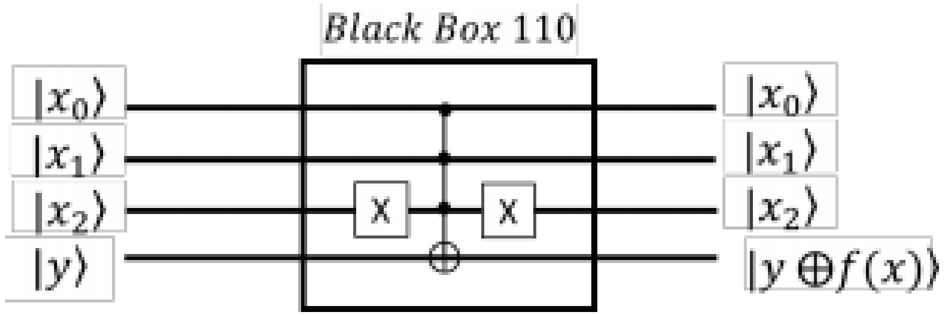
(May 10 Session 7)



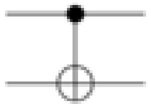
x	$y = f(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	1
111	0



x	$y = f(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	1
111	0

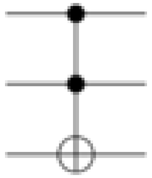


Controlled Not (CNOT, CX)



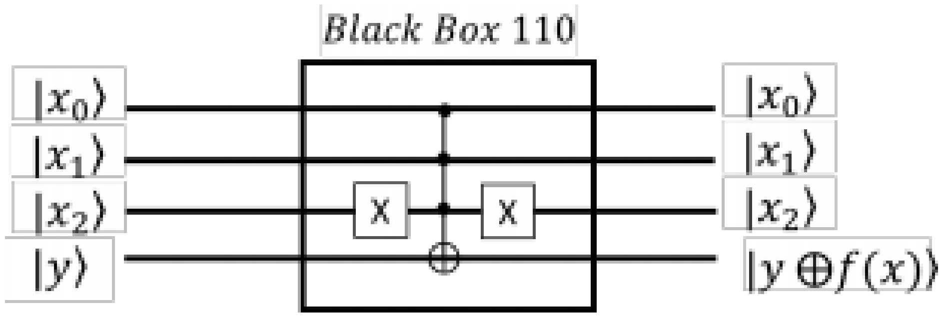
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli (CCNOT, CCX, TOFF)

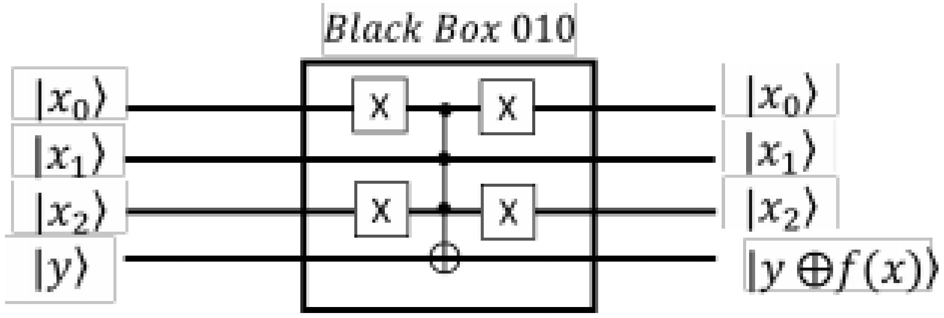


$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

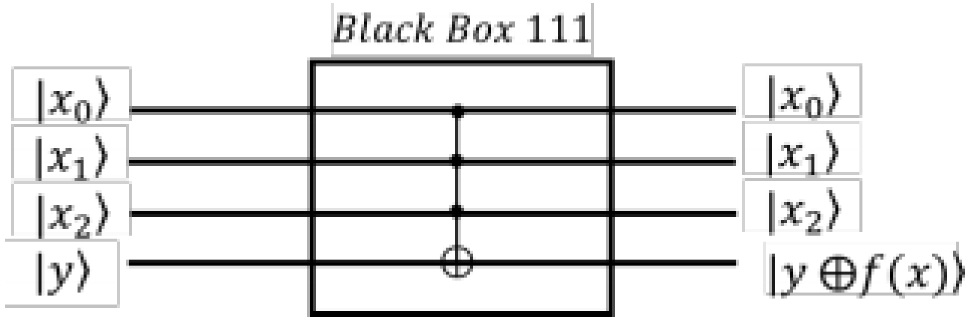
x	$y = f(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	1
111	0

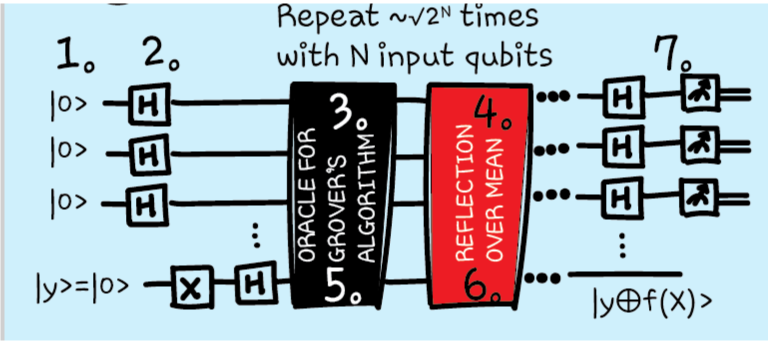


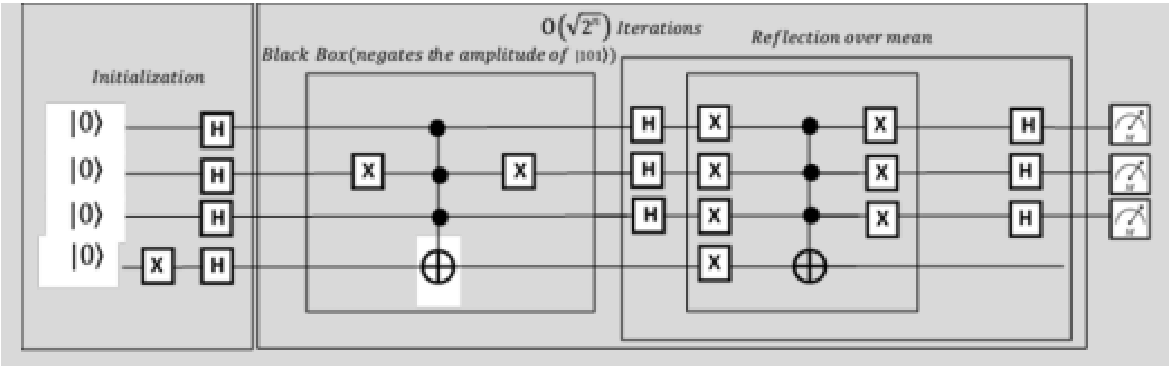
x	$y = f(x)$
000	0
001	0
010	1
011	0
100	0
101	0
110	0
111	0

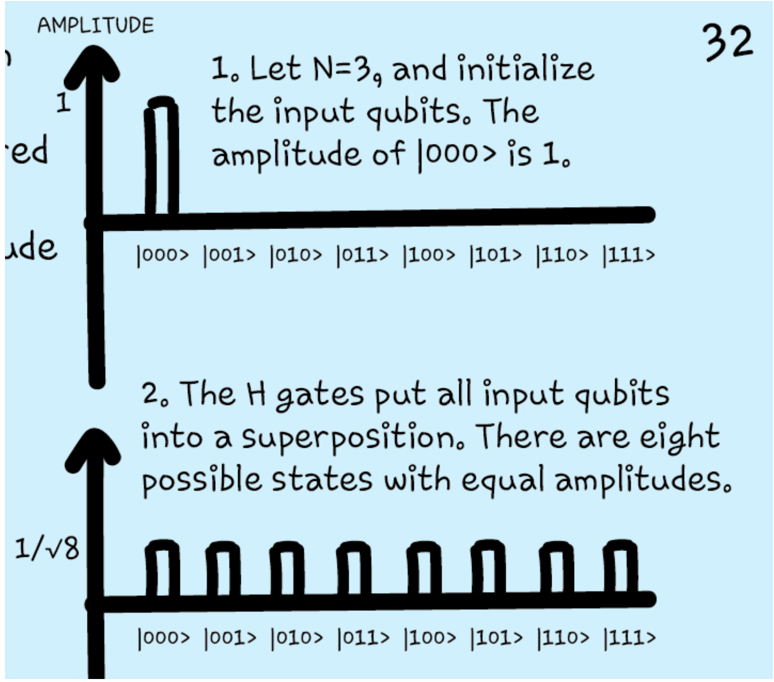


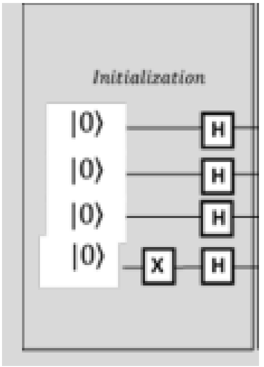
x	$y = f(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	0
111	1

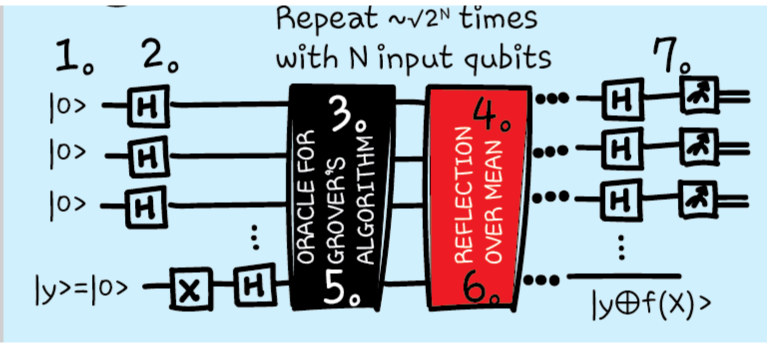


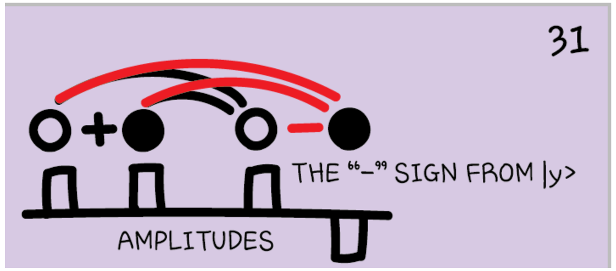

Repeat ~√2^N times
with N input qubits
1.
2.
3.
4.
5.
6.
7.
|0>
|0>
|0>
|y>=|0>
ORACLE FOR GROVER'S ALGORITHM.
REFLECTION OVER MEAN
|y⊕f(X)>


O(√2^n) Iterations
Initialization
Black Box(negates the amplitude of |101⟩)
Reflection over mean
|0⟩
|0⟩
|0⟩
|0⟩

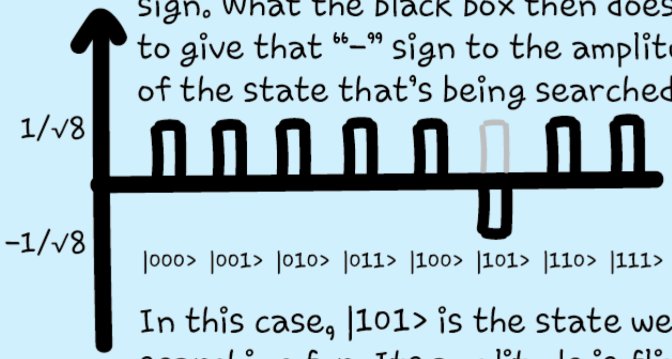

AMPLITUDE
32
1
1. Let N=3, and initialize the input qubits. The amplitude of |000> is 1.
|000> |001> |010> |011> |100> |101> |110> |111>
2. The H gates put all input qubits into a superposition. There are eight possible states with equal amplitudes.
1/√8
|000> |001> |010> |011> |100> |101> |110> |111>


Initialization
|0⟩
|0⟩
|0⟩
|0⟩

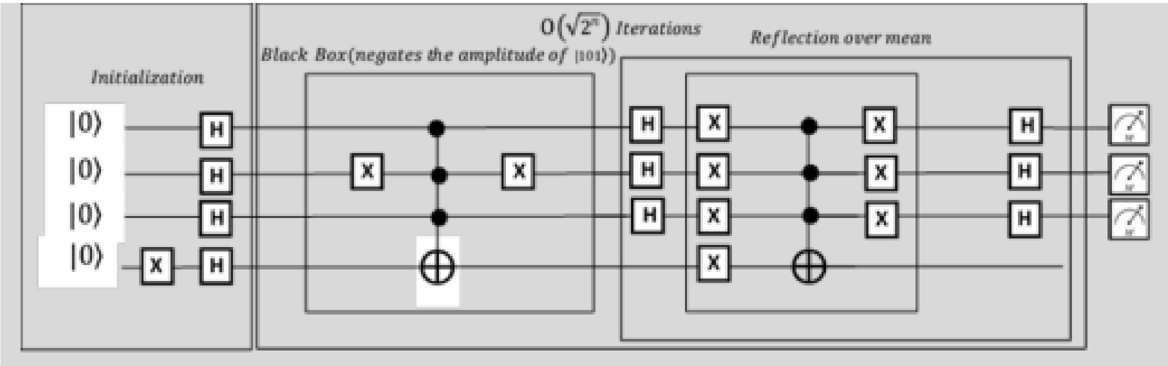


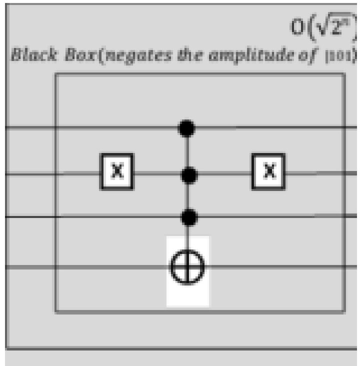


3. The last qubit |y> introduces a "–" sign. What the black box then does is to give that "–" sign to the amplitude of the state that's being searched for.

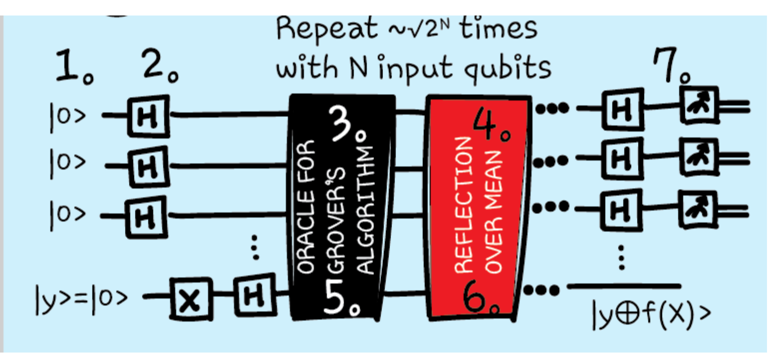


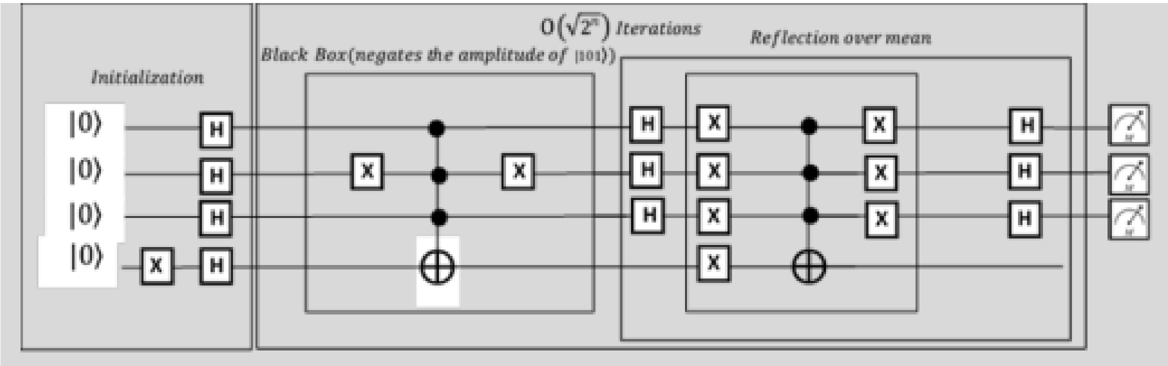
In this case, |101> is the state we are searching for. Its amplitude is flipped over the horizontal axes.

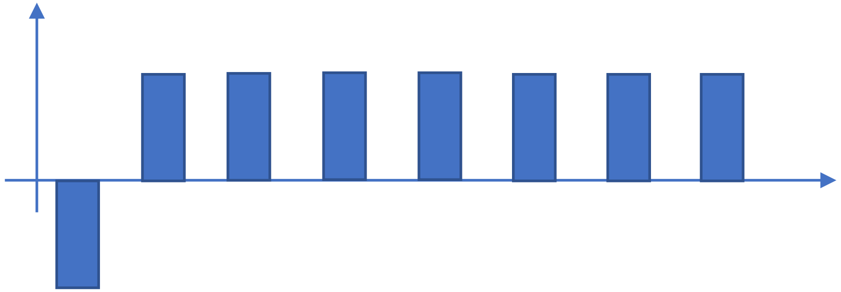


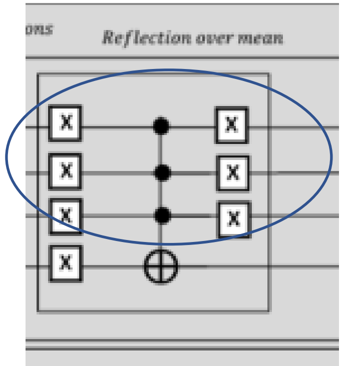


x	$y = f(x)$
000	0
001	0
010	0
011	0
100	0
101	1
110	0
111	0



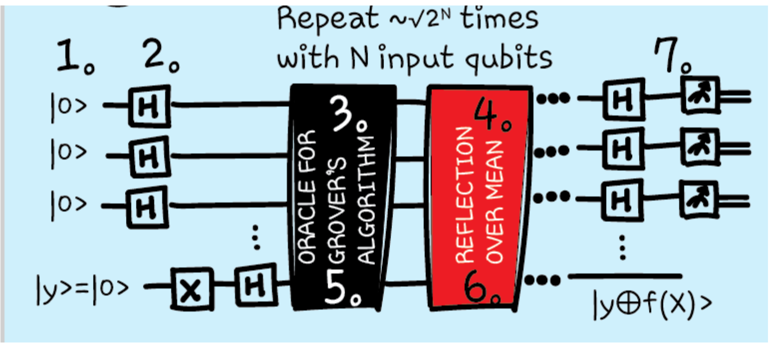


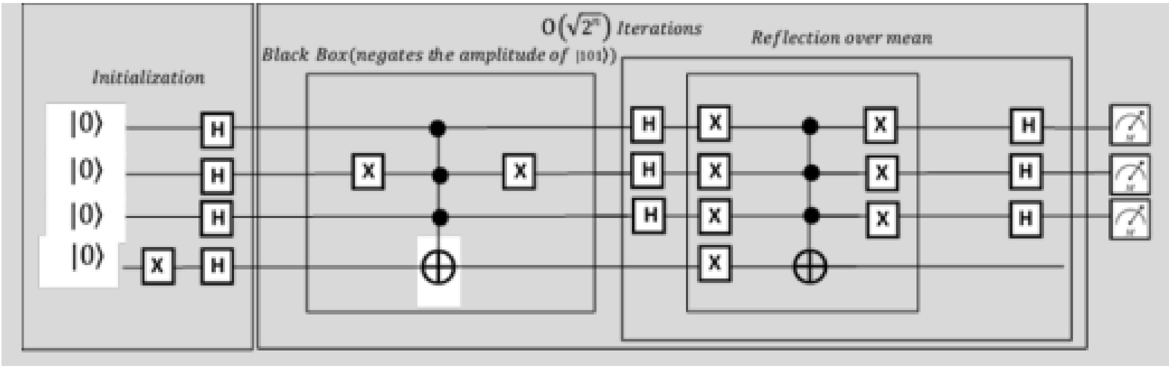


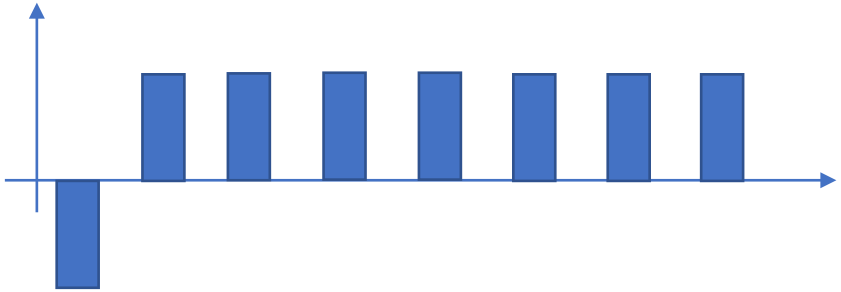


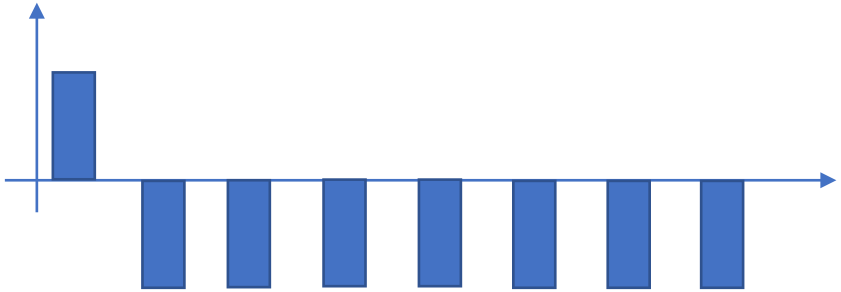
$$-a_0\,|000\rangle \otimes \left(-\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}\right) - a_1|001\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_2|010\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_3|011\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_4|100\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_5|101\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_6|110\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_7|111\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$

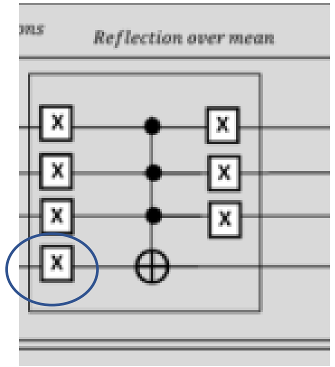
$$= (a_0|000\rangle - a_1|001\rangle - a_2|010\rangle - a_3|011\rangle - a_4|100\rangle - a_5|101\rangle - a_6|110\rangle - a_7|111\rangle) \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$





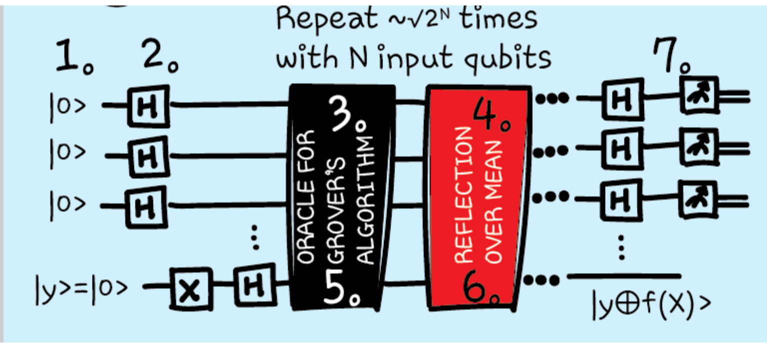


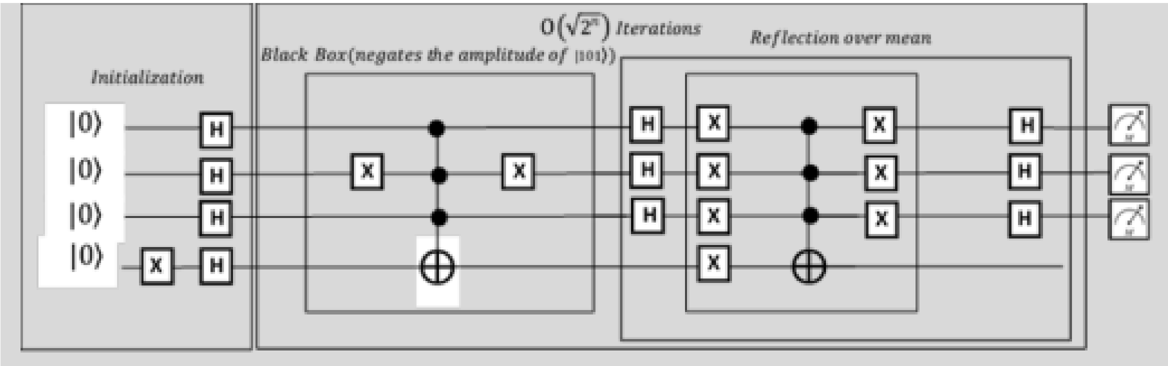


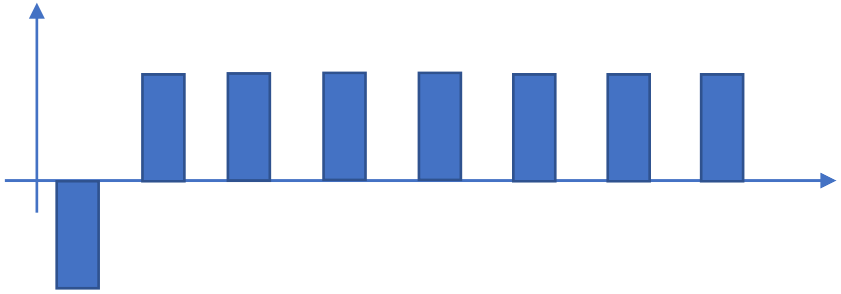


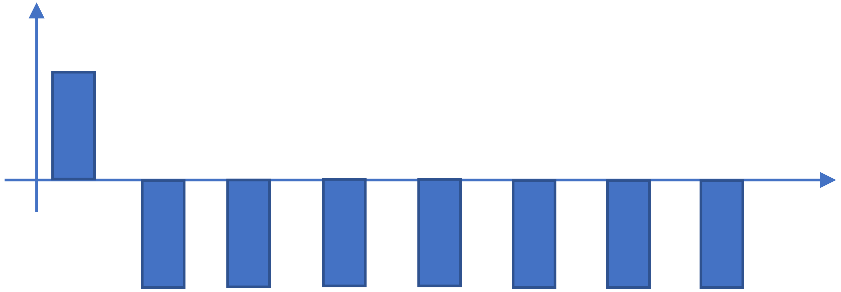
$$-a_0\,|000\rangle \otimes \left(-\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}\right) - a_1|001\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_2|010\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_3|011\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_4|100\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_5|101\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_6|110\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_7|111\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$

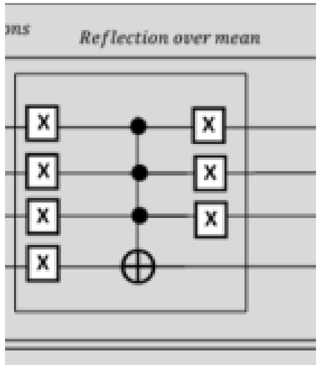
$$= (a_0|000\rangle - a_1|001\rangle - a_2|010\rangle - a_3|011\rangle - a_4|100\rangle - a_5|101\rangle - a_6|110\rangle - a_7|111\rangle) \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$





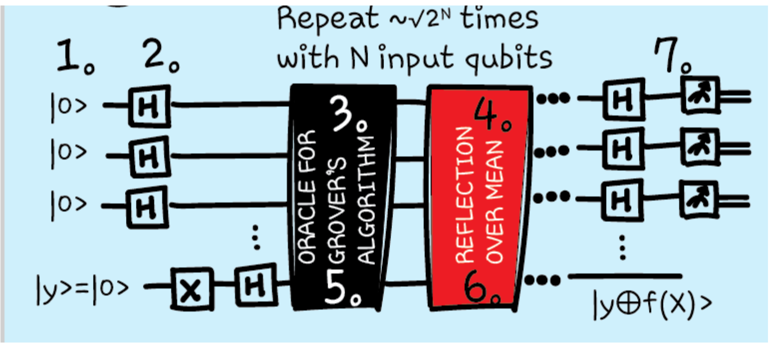


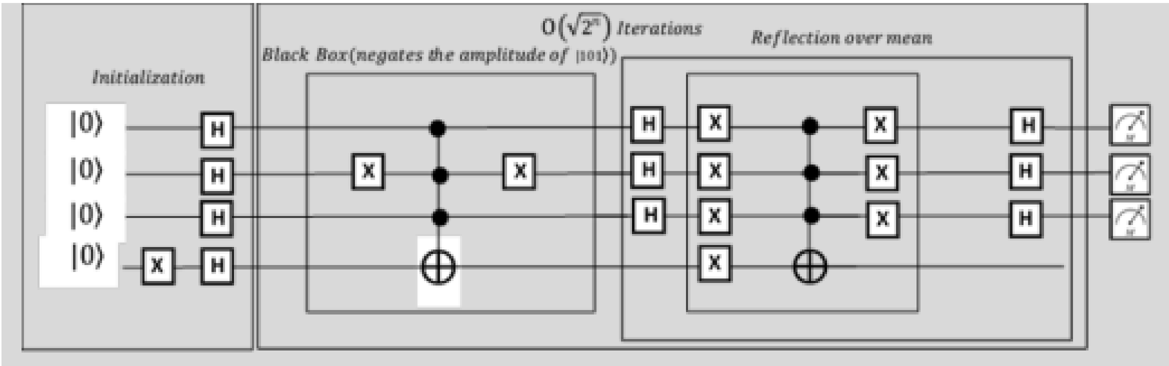


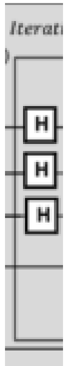


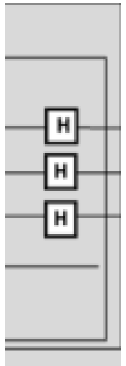
$$-a_0\,|000\rangle \otimes \left(-\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}\right) - a_1|001\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_2|010\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_3|011\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_4|100\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_5|101\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_6|110\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right) - a_7|111\rangle \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$

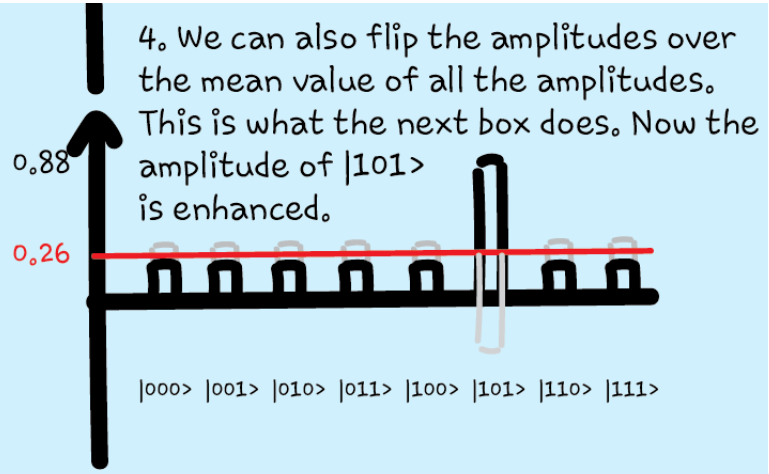
$$= \left(a_0|000\rangle - a_1|001\rangle - a_2|010\rangle - a_3|011\rangle - a_4|100\rangle - a_5|101\rangle - a_6|110\rangle - a_7|111\rangle\right) \otimes \left(\frac{|0\rangle}{\sqrt{2}}-\frac{|1\rangle}{\sqrt{2}}\right)$$

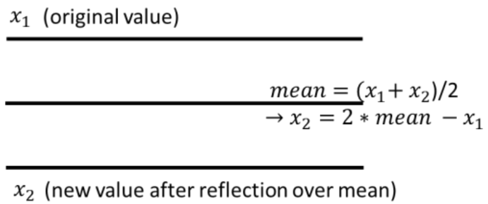

Repeat ~√2ᴺ times
with N input qubits
1. 2.
7.
|0>
|0>
|0>
|y>=|0>
3.
ORACLE FOR GROVER'S ALGORITHM
5.
4.
REFLECTION OVER MEAN
6.
|y⊕f(X)>

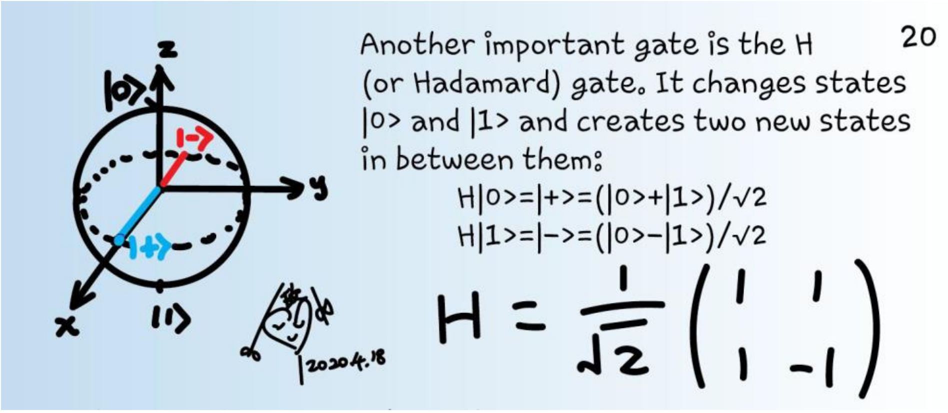

O(√2ⁿ) Iterations
Black Box(negates the amplitude of
Reflection over mean
Initialization
|0⟩
|0⟩
|0⟩
|0⟩

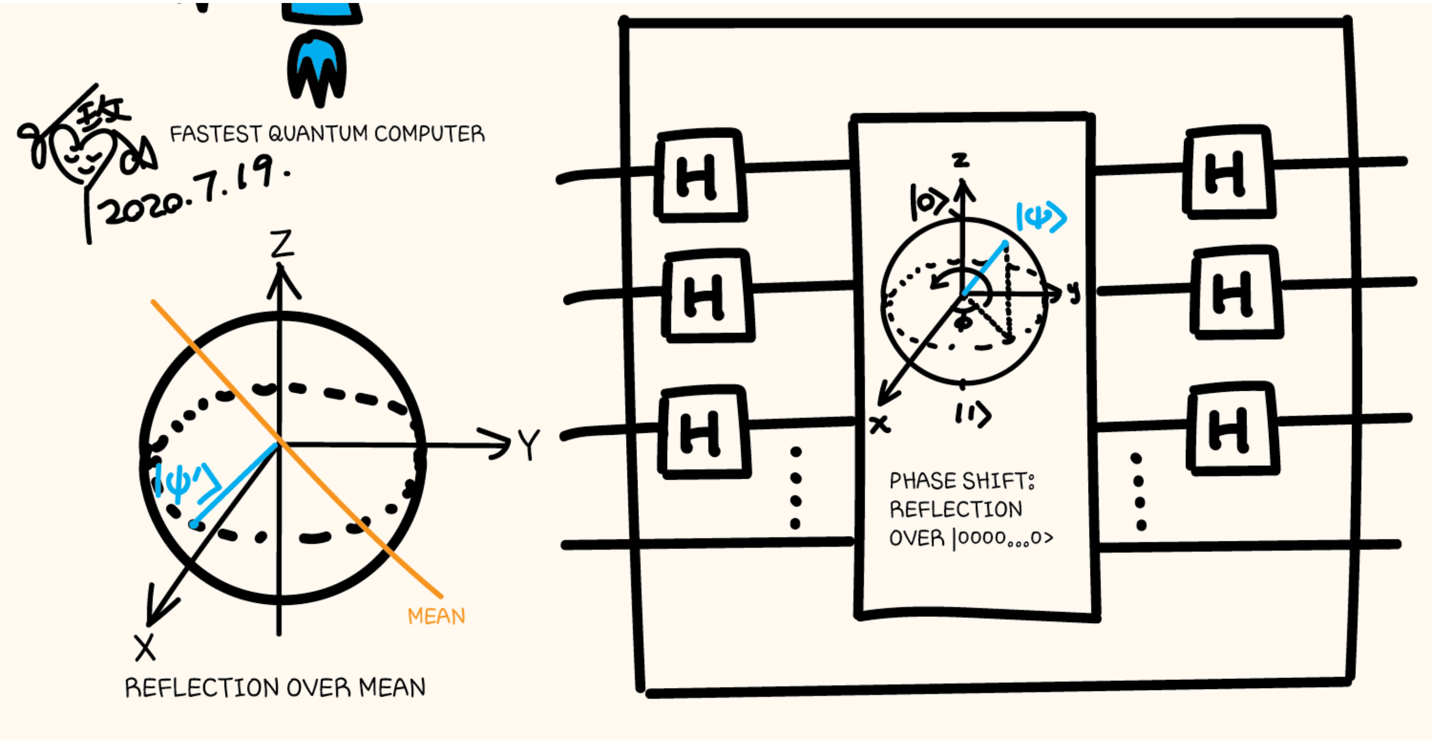


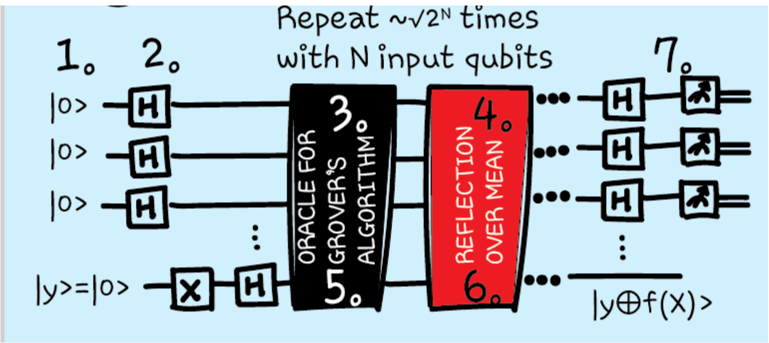


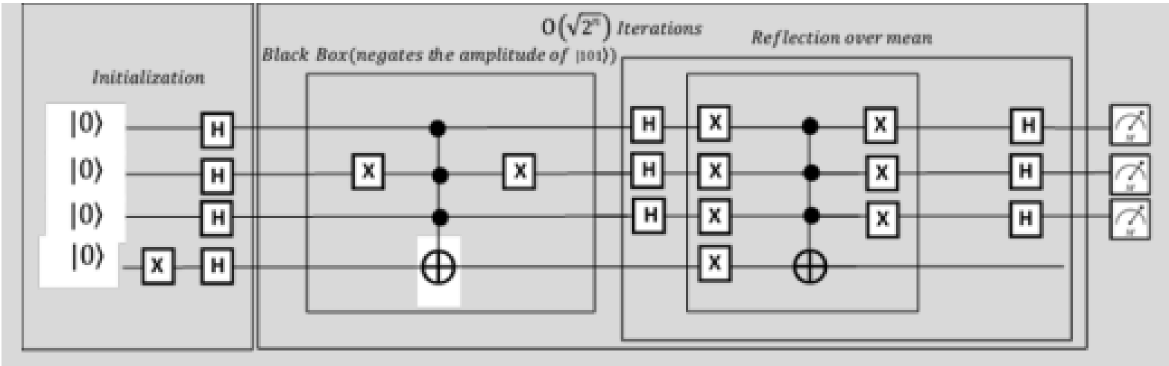

4. We can also flip the amplitudes over the mean value of all the amplitudes. This is what the next box does. Now the amplitude of |101>
is enhanced.
0.88
0.26
|000> |001> |010> |011> |100> |101> |110> |111>

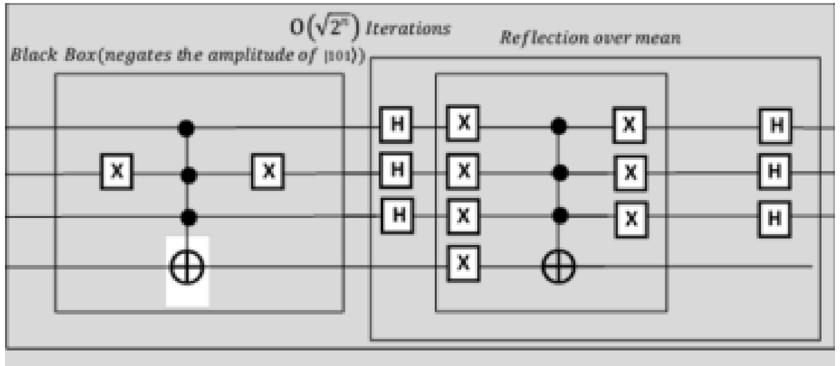

x_1 (original value)
$mean = (x_1 + x_2)/2$
$\rightarrow x_2 = 2 * mean - x_1$
x_2 (new value after reflection over mean)

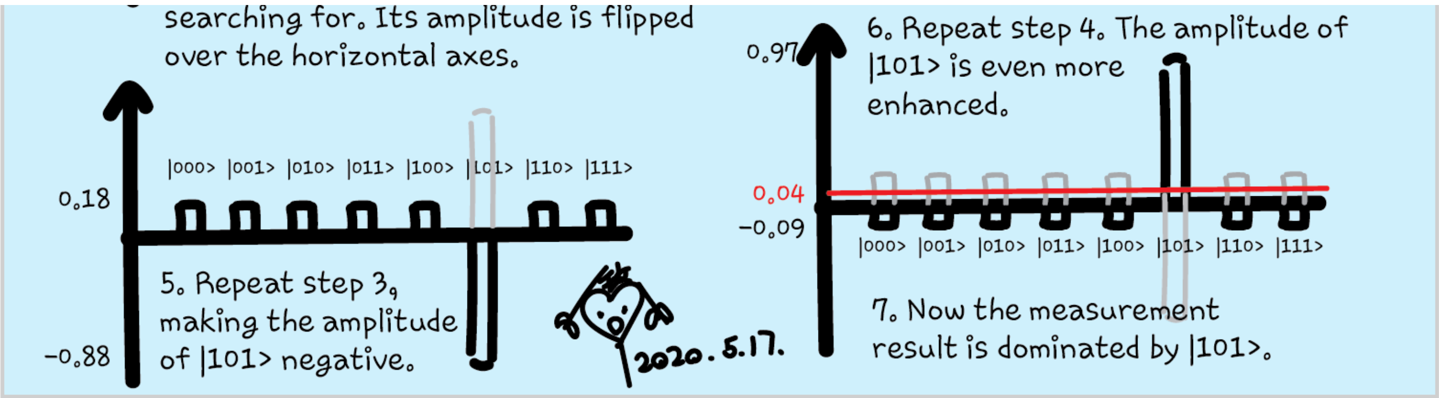

Another important gate is the H
(or Hadamard) gate. It changes states
|0> and |1> and creates two new states
in between them:
H|0>=|+>=(|0>+|1>)/√2
H|1>=|->=(|0>-|1>)/√2
H = 1/√2 (1 1 / 1 -1)
z
|0>
|->
|+>
y
x
|1>
2020.4.18


FASTEST QUANTUM COMPUTER
2020.7.19.
Z
Y
X
|ψ'⟩
MEAN
REFLECTION OVER MEAN
H
z
|0⟩
|ψ⟩
y
x
|1⟩
PHASE SHIFT:
REFLECTION
OVER |0000...0⟩


Repeat ~√2ᴺ times
with N input qubits
1.
2.
7.
|0>
|0>
|0>
|y>=|0>
ORACLE FOR GROVER'S ALGORITHM.
3.
5.
REFLECTION OVER MEAN
4.
6.
|y⊕f(X)>


Initialization
Black Box(negates the amplitude of |101⟩)
O(√2ⁿ) Iterations
Reflection over mean
|0⟩
|0⟩
|0⟩
|0⟩


Black Box(negates the amplitude of |101⟩)
O(√2ⁿ) Iterations
Reflection over mean


searching for. Its amplitude is flipped
over the horizontal axes.
0.18
-0.88
|000> |001> |010> |011> |100> |101> |110> |111>
5. Repeat step 3,
making the amplitude
of |101> negative.
2020.5.17.
6. Repeat step 4. The amplitude of
|101> is even more
enhanced.
0.97
0.04
-0.09
|000> |001> |010> |011> |100> |101> |110> |111>
7. Now the measurement
result is dominated by |101>.

Quantum katas

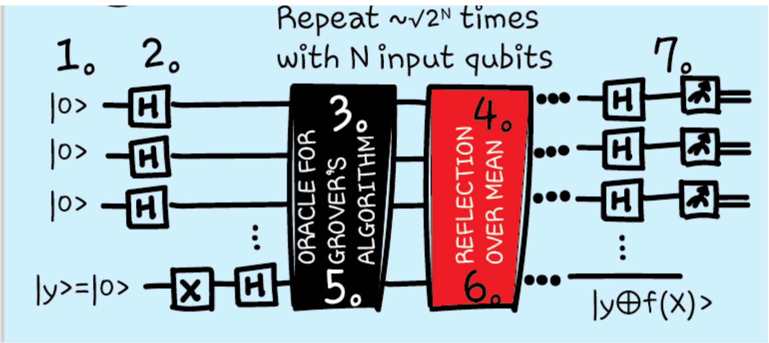
- Set up Grover's algorithm from scratch — https://github.com/microsoft/QuantumKatas/tree/master/GroversAlgorithm
- Use Grover's algorithm — https://github.com/microsoft/QuantumKatas/tree/master/tutorials/ExploringGroversAlgorithm
- Visualize Grover's algorithm — https://github.com/microsoft/QuantumKatas/tree/master/GraphColoring
- Decorating the Christmas tree using Grover's search — https://github.com/tcNickolas/MiscQSharp/tree/master/DecoratingTheTree

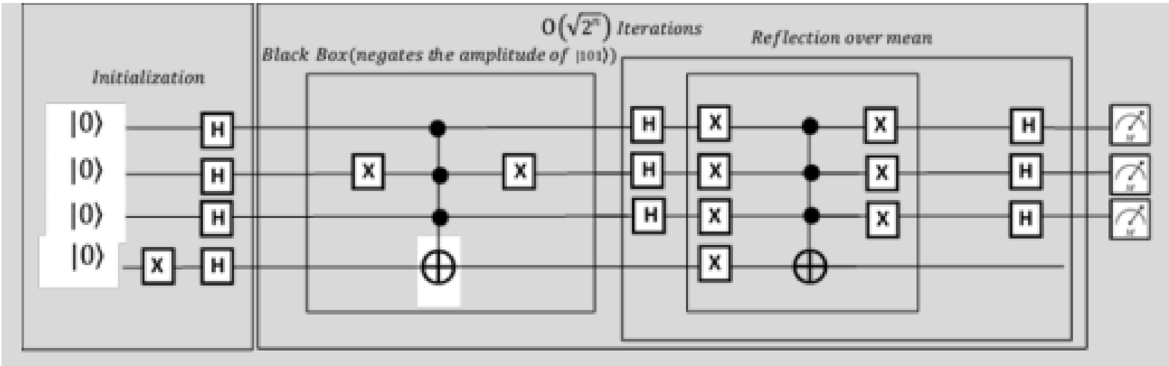
Q# exercise:

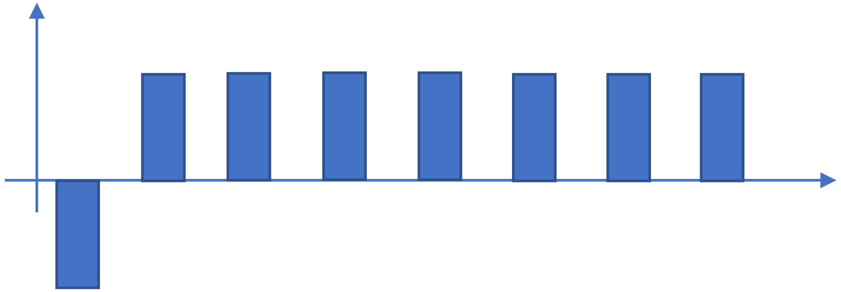
Quantum Katas

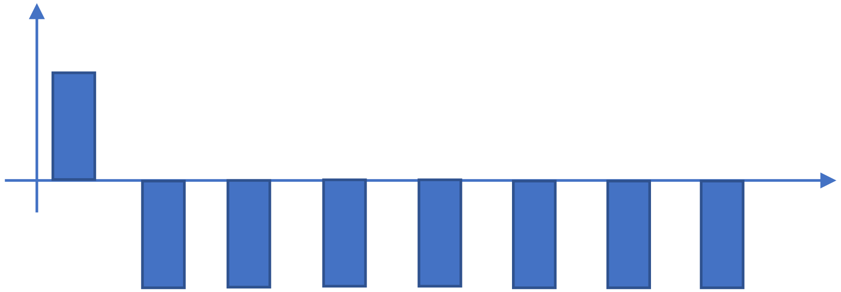
https://github.com/Microsoft/QuantumKatas

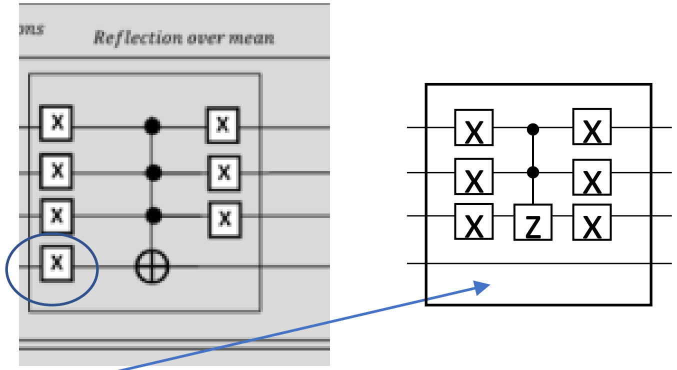
- **GroversAlgorithm**
  - Task 1.1, 2.1-2.3



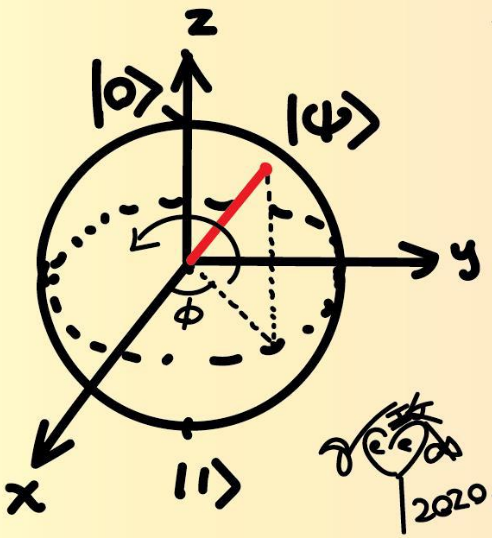








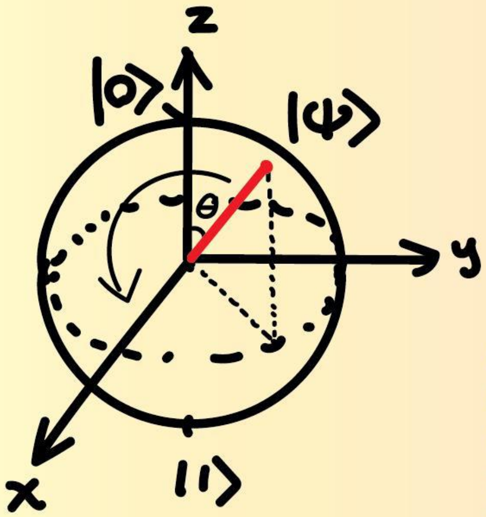
Introduce the “-” sign



To change the phase φ, we have a commonly used gate, Z, which rotates about the z-axis by 180°.

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Similarly, the X gate rotates about the x-axis by 180°, rotating the angle θ e.g. X|0> = |1>, X|1> = |0>.



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

We have seen in page 18 the two matrices for changing φ and θ in arbitraty amounts. They form a universal gate set - they can put a state anywhere on the Bloch Sphere. The gates Z and X are special cases of them.

Questions

- Post in chat or on Hackaday project
  https://hackaday.io/project/168554-quantum-computing-through-comics

- FAQ: Past Recordings on Hackaday project or my YouTube https://www.youtube.com/c/DrKittyYeung